Various Cluster Radioactivities above Magic Nuclei

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Abstract

We present parameter-free tunneling calculations for various cluster radioactivities including the diproton decays of atomic nuclei. An uniform folded cluster potential has been suggested that is based on a self-consistent mean-field model, with the folding factor determined using the quantization conditions of the quasibound cluster state. We have investigated the α-particle and heavier-cluster decays of trans-^{100}Sn and trans-^{208}Pb nuclei, and the observed diproton emission from the proton drip-line nucleus ^{16}Ne, showing the overall reasonable descriptions of cluster radioactivities with calculated half-lives agreeing well with experimental data. We have also predicted the properties of yet unobserved cluster decays of the exotic nuclei ^{112,114}Ba, ^{104}Te and ^{38}Ti.

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The charged-particle emission is one of the most important decay modes of atomic nuclei. Glancing at the chart of nuclides, one can find that almost all observed proton-rich exotic nuclei starting from $A \sim 150$ have α radioactivities. α decays occur more widely in nuclei heavier than lead isotopes. By a simple picture, the α decay can be understood as a process of the α particle being formed in the mother nucleus and emitted tunneling through a Coulomb barrier which is created due to the Coulomb interaction between the cluster and the remaining nucleons (i.e. the daughter system). The study of α decays has provided rich information about the structures of nuclei.

The emissions of heavier clusters, such as ^{14}C, ^{20}O, ^{24}Ne, ^{28}Mg and ^{32}Si, have been well established experimentally in trans-lead nuclei decaying into daughters around the doubly magic nucleus of ^{208}Pb [1, 2, 3]. A second island of heavy-cluster radioactivities was predicted [4] in trans-tin nuclei decaying into daughters close to the doubly magic nucleus of ^{100}Sn. Intense studies, particularly on the most promising case of the ^{12}C emission from the proton drip-line nucleus ^{114}Ba decaying into ^{102}Sn, have been made both experimentally [5, 6, 7, 8] and theoretically [9, 10, 11, 12]. Experiments have not pinned down the observation of the ^{12}C decay of ^{114}Ba. Theoretically, the predictions of the partial half-life can be different by several orders of magnitude [9, 10, 11, 12]. However, the recent experiment [8] has derived the Q value for the possible ^{12}C decay of ^{114}Ba, which is very important for the theoretical prediction because the calculated half-life depends dramatically on the Q value. At the proton drip line, another exciting phenomenon is the diproton radioactivity of nuclei. Modern facilities have been opening new perspectives for the study of exotic decays.

The decay process of a charged cluster can be treated with the Wentzel-Kramers-Brillouin (WKB) approach. In the present work, we have investigated cluster decays from ground states to ground states in even-even nuclei. In such decay, the cluster does not carry any angular momentum, i.e. $L = 0$. The decay width can be written as (see, e.g., [13])

$$\Gamma = PF\frac{\hbar^2}{4\mu}\exp\Big[-2\int_{r_1}^{r_2} dr k(r)\Big], \tag{1}$$

where P is the preformation probability of the cluster being formed in the mother, and μ is the reduced mass of the cluster-daughter system. The normalization factor F is given by

$$F\int_0^{r_1} dr\frac{1}{k(r)}\cos^2\Big[\int_0^{r} dr'k(r') - \frac{\pi}{4}\Big] = 1, \tag{2}$$

and the wave number $k(r)$ is defined by

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|Q - V(r)|}, \tag{3}$$

where Q is the decay energy and $V(r)$ is the cluster potential in which the cluster moves. r_1 and r_2 are the classical inner and outer turning points, respectively, obtained by $V(r) = Q$. Then the half-life of the cluster decay is obtained by $T_{1/2} = \hbar \ln 2/\Gamma$.

In the above method, the determination of the cluster potential $V(r)$ is a key step for the decay calculation. Phenomenologic potentials by Buck *et al.* [13] and potentials based on effective nucleon-nucleon interactions [14, 15, 16] have been successfully applied to the WKB calculations of the charged-particle decays of nuclei. The cluster potential can be written as

$$V(r) = V_{\mathrm{N}}(r) + V_{\mathrm{C}}(r), \tag{4}$$

where $V_{\mathrm{N}}(r)$ and $V_{\mathrm{C}}(r)$ are the nuclear potential and the Coulomb potential, respectively, between the cluster and the daughter system. In the present work, the nuclear potential is constructed by nucleon potentials with multiplying a folding factor λ as follows

$$V_{\mathrm{N}}(r) = \lambda(N_{\mathrm{c}} v_{\mathrm{n}}(r) + Z_{\mathrm{c}} v_{\mathrm{p}}(r)), \tag{5}$$

where N_{c} and Z_{c} are the neutron and proton numbers of the cluster, respectively; $v_{\mathrm{n}}(r)$ and $v_{\mathrm{p}}(r)$ are the neutron and proton potentials (excluding the Coulomb potential) respectively, obtained from a self-consistent mean-field model. Folding processes have been employed in other forms of α-nucleus potentials based on effective nucleon-nucleon interactions [14, 15, 16]. The Coulomb potential $V_{\mathrm{C}}(r)$ is well defined physically and should not be folded. We have approximated the Coulomb potential by $V_{\mathrm{C}}(r) = Z_{\mathrm{c}} v_{\mathrm{c}}(r)$, where $v_{\mathrm{c}}(r)$ is the proton Coulomb potential obtained from the mean-field calculation.

In the tunneling model, the Bohr-Sommerfeld quantization condition defines a relation between the potential $V(r)$ and decay energy Q for a quasibound cluster state. We have used the condition to determine the folding factor λ at a given Q value. It will be seen that thus determined folded α-nucleus potential leads to a realistic volume integral (per nucleon pair) which is obtained from α-particle scatterings experimentally [14, 15]. The Bohr-Sommerfeld condition is given by [13]

$$\int_0^{r_1} dr \sqrt{\frac{2\mu}{\hbar^2}|Q - V(r)|} = (2n+1)\frac{\pi}{2} = (G+1)\frac{\pi}{2}, \tag{6}$$

where n is the node number in the $L = 0$ radial wave function of the relative motion of the cluster in the potential, and G is the oscillator quantum number with $G = 2n$ [13]. The quantum number G can be determined by the Wildermuth condition, $G = \sum_{i=1}^{\mathrm{A_c}} g_i$ [15], where $\mathrm{A_c}$ is the mass number of the cluster and g_i is the oscillator quantum number of a cluster nucleon orbiting in the mean field. Actually, g_i is just the main quantum number in the Nilsson labelling. For the lowest-energy cluster decay, cluster nucleons should occupy orbits immediately above the Fermi levels of the daughter nucleus. For trans-^{100}Sn nuclei, for example, cluster nucleons occupy the $g = 4$ orbits, making $G = 4\mathrm{A_c}$. In trans-^{208}Pb nuclei, the orbits occupied by cluster nucleons have $g = 5$ for protons and $g = 6$ for neutrons, leading to $G = 5Z_\mathrm{c}+6N_\mathrm{c}$. Hence, the combined use of the Bohr-Sommerfeld and Wildermuth quantization conditions can determine the folding factor λ.

Now it has been seen that the present method does not bring any extra adjustable parameter. The parameter-free calculation is particularly useful for exotic nuclei where experimental data are lacking. Calculations based on self-consistent mean-field models should in principle give more consistent descriptions on both the structures and decays of nuclei. It has been known that mean-field models can in general produce the structure properties of exotic nuclei [17]. Mean-field potentials, due to their self-consistent interdependence of proton and neutron densities that are fed back into potentials, automatically contain a dependence on nucleon numbers, i.e. an isospin dependence, in a self-consistent manner [18].

In a very recent work [18], a form of the α-nucleus potential by $V(r) = 2v_\mathrm{p}(r) + 2v_\mathrm{n}(r)$ has been used with $v_\mathrm{p}(r)$ and $v_\mathrm{n}(r)$ calculated by mean-field models. However, an extra parameter determined by fitting decay data is needed for α-decay calculations [18]. Also, it will be seen that the α-nucleus potential without folding gives a too deep well, leading to a too large volume integral. Hence, the folding process is also to reduce the depth of the cluster potential.

In our calculations, a preformation factor of $P = 1$ has been assumed for the various cluster decays of even-even nuclei, as suggested in Refs. [11, 13]. The potentials $v_\mathrm{n}(r)$, $v_\mathrm{p}(r)$ and $v_\mathrm{c}(r)$ have been calculated in the spherical daughter system using the Skyrme-Hartree-Fock (SHF) approach with the SkI4 force that has been developed with good isospin properties [19]. Actually, we found that different Skyrme forces in general lead to similar results for cluster decay calculations. Spherical shapes allow us to perform simple one-dimension tunneling calculations, as in Refs. [13, 14, 15, 16, 18, 20]. Compared with the

preformation factor, the Q value is a much more crucial quantity that affects the calculated decay half-life, since the half-life is exponentially dependent on the Q value. Therefore, we have adopted experimental Q values that can be obtained from the measured masses (binding energies) of mothers, daughters and clusters.

As a test, we have calculated the α-decay property of the spherical nucleus ^{212}Po that has been shown to have a significant α-cluster structure outside the core of the doubly magic nucleus ^{208}Pb [21]. Our WKB calculation leads to $T^{\alpha}_{1/2} = 89$ ns against the experimental half-life of 299 ± 2 ns [22]. The ratio between the calculated and measured half-lives gives a preformation factor of 0.3 agreeing with the value of 0.3 calculated by the shell-cluster model [21]. The folding factor λ is determined to be 0.595. Such a folded α-nucleus potential gives a rather reasonable volume integral of $J_{\rm v} = 325$ MeV fm^3 which is in a good agreement with the experimental $J_{\rm v} \approx 300 - 350$ MeV fm^3 for a wide range of nuclei [15], obtained from α-particle scattering measurements. The α-nucleus potential without folding [18] leads to a too large volume integral of ~ 550 MeV fm^3.

With the development of experimental techniques, more and more exotic nuclei far from the stability are being produced. Their decay properties are attracting great interest experimentally and theoretically. In the present work, we have investigated the cluster decays of trans-^{100}Sn and trans-^{208}Pb nuclei in which α-particle and heavier-cluster radioactivities have been observed or expected. Table I shows calculated α-decay properties. The calculated half-lives agree with experimental data within a factor of ≈ 2, except ^{114}Ba for which the experiment of Ref. [6] gave a partial half-life of $T^{\alpha}_{1/2} \geq 1.2 \times 10^2$ s. As predictions, we have also calculated the α-decay half-lives of the unknown proton drip-line nuclei ^{104}Te and ^{112}Ba, shown in Table I. In the calculations, theoretical Q values given by Möller *et al.* [23] have been adopted. However, a change of 1 MeV in the Q value can lead to a variation of ≈ 3 orders of magnitude in the half-life calculation. From Table I, it can be seen that the folding factors of the α-nucleus potentials are quite stable at $\lambda \sim 0.5$ for trans-^{100}Sn nuclei and $\lambda \sim 0.6$ for trans-^{208}Pb nuclei. Resulting volume integrals are $J_{\rm v} \sim 290$ and ~ 330 MeV fm^3 for the trans-^{100}Sn and trans-^{208}Pb nuclei, respectively.

Heavier-cluster radioactivities in trans-^{208}Pb have been observed well [1, 2, 3]. Theoretical investigations have also been made [11, 24]. Our parameter-free calculations for the partial half-lives of the heavy-cluster decays agree with observations within one order of magnitude, shown in Table II. The determined folding factors have similar values to that

TABLE I: Calculated half-lives of the α decays in trans-^{100}Sn and trans-^{208}Pb nuclei, compared with experimental half-lives obtained from evaluated data in Ref. [22]. For ^{114}Ba, another experimental half-life [6] is given for comparison.

Emitter	Q [22] (MeV)	G	λ	$T^{cal}_{1/2}$ (s)	$T^{expt}_{1/2}$ (s)
^{104}Te	6.12^a	16	0.529	7.3E−11	
^{106}Te	4.290	16	0.521	1.4E−4	(7.0±2.0)E−5
^{108}Te	3.445	16	0.484	4.9E+0	(4.3±0.2)E+0
^{110}Te	2.723	16	0.472	1.6E+6	(6.2±0.3)E+5
^{110}Xe	3.885	16	0.435	2.1E−1	(4.8±3.0)E−1
^{112}Xe	3.330	16	0.435	6.1E+2	(3.0±0.9)E+2
^{112}Ba	4.26^a	16	0.539	5.4E−2	
^{114}Ba	3.530	16	0.439	7.1E+2	(5.9±2.6)E+1
					≥1.2E+2 [6]
^{222}Ra	6.679	22	0.609	2.5E+1	(3.80±0.05)E+1
^{224}Ra	5.789	22	0.611	2.9E+5	(3.16±0.03)E+5
^{226}Ra	4.871	22	0.612	5.8E+10	(5.05±0.02)E+10
^{228}Th	5.520	22	0.611	7.8E+7	(6.03±0.01)E+7
^{230}Th	4.770	22	0.611	3.6E+12	(2.38±0.01)E+12
^{232}U	5.414	22	0.609	3.2E+9	(2.17±0.01)E+9
^{234}U	4.858	22	0.609	1.1E+13	(7.75±0.01)E+12
^{236}Pu	5.867	22	0.605	8.4E+7	(9.02±0.02)E+7
^{238}Pu	5.593	22	0.604	2.5E+9	(2.77±0.00)E+9

[a]Theoretical value from [23].

of corresponding α-nucleus potentials. For the ^{12}C radioactivity in ^{114}Ba, the recent experiment has derived the decay energy of $Q = 19.00 \pm 0.04$ MeV [8]. With the experimental Q value, we have calculated a partial half-life of $T_{1/2} = 1.5 \times 10^{10}$ s and a branching ratio of $b = 4.7 \times 10^{-8}$ ($b = \Gamma(^{12}\mathrm{C})/\Gamma(\alpha)$). The experiment by Guglielmetti *et al.* [7] has estimated a partial half-life of $T_{1/2} \geq 1.2 \times 10^4$ s and a branching ratio of $b \leq 3.4 \times 10^{-5}$ for the ^{12}C

TABLE II: Partial half-lives and branching ratios (relative to the α decay) of the heavy-cluster decays in trans-^{208}Pb and trans-^{100}Sn nuclei. For trans-^{208}Pb nuclei, measured partial half-lives have been compiled in Ref. [11], and experimental branching ratios and Q values have been obtained from Ref. [22]. For trans-^{100}Sn nuclei, numbers in square brackets indicate references from which the adopted values come.

Decay	Q (MeV)	G	λ	$T^{\mathrm{cal}}_{1/2}$ (s)	b^{cal}	$T^{\mathrm{expt}}_{1/2}$ (s)	b^{expt}
^{222}Ra(^{14}C)	33.05	78	0.540	4.3E+10	5.8E−10	(1.01±0.14)E+11	(3.0±1.0)E−10
^{224}Ra(^{14}C)	30.54	78	0.540	2.0E+15	1.5E−10	(8.25±2.22)E+15	(4.0±1.2)E−11
^{226}Ra(^{14}C)	28.20	78	0.544	2.0E+20	2.9E−10	(2.21±0.96)E+21	(2.6±0.6)E−11
^{228}Th(^{20}O)	44.72	112	0.553	7.1E+20	1.1E−13	(5.29±1.01)E+20	(1.1±0.2)E−13
^{230}Th(^{24}Ne)	57.76	132	0.563	1.2E+24	3.0E−12	(4.10±0.95)E+24	(5.6±1.0)E−13
^{232}U(^{24}Ne)	62.31	134	0.556	7.3E+19	4.4E−11	(2.50±0.30)E+20	(8.9±0.7)E−12
^{234}U(^{28}Mg)	74.11	154	0.567	7.8E+24	1.4E−12	(5.50±1.00)E+25	(1.4±0.3)E−13
^{236}Pu(^{28}Mg)	79.67	156	0.561	3.6E+20	2.3E−13	4.7E+21	2E−14
^{238}Pu(^{32}Si)	91.19	176	0.571	3.0E+25	8.3E−17	(1.89±0.68)E+25	1.4E−16
^{114}Ba(^{12}C)	19.00 [8]	48	0.498	1.5E+10	4.7E−8	≥1.2E+4 [7]	≤3.4E−5 [7]
^{112}Ba(^{12}C)	21.46 [10]	48	0.499	9.5E+4	5.7E−7		
^{112}Ba(^{12}C)	23.17 [23]	48	0.496	6.7E+1	8.1E−4		

decay of ^{114}Ba. The nucleus ^{112}Ba has not been known experimentally. We have used the theoretical decay energy of $Q = 23.17$ MeV given by Möller *et al.* [23] to calculate the partial half-life of the ^{12}C decay of ^{112}Ba, giving $T_{1/2} = 67$ s. Kumar *et al.* gave another calculated value of $Q = 21.46$ MeV [10], leading to a half-life of 9.5×10^4 s by our calculation.

Around the proton drip line, the diproton radioactivity is another exciting challenge in both experiment and theory. Experiments have observed a few examples of diproton emissions from light proton drip-line nuclei [22]. Nazarewicz *et al.* [20] have investigated diproton radioactivities around the doubly magic nucleus ^{48}Ni in the framework of the WKB method with the form of diproton potential by $V_{2\mathrm{p}}(r) = 2v_{\mathrm{p}}(r)$ (They included the Coulomb potential in $v_{\mathrm{p}}(r)$). The proton potential $v_{\mathrm{p}}(r)$ was calculated using various mean-field models [20]. They compared the depths of potentials, resulting in a modification on the term of $|Q - V_{2\mathrm{p}}|$ by multiplying an effective mass of $m^*/m < 1$ [20]. Similarly to cluster decays

discussed above, we have approximated the diproton potential by $V_{2\mathrm{p}}(r) = \lambda \times 2v_{\mathrm{p}}(r) + 2v_{\mathrm{c}}(r)$ (Note that our $v_{\mathrm{p}}(r)$ excludes the Coulomb potential). Again, the half-life calculation of the diproton decay is dramatically dependent on the decay energy of $Q_{2\mathrm{p}}$.

The diproton emission from the proton drip-line nucleus ^{16}Ne decaying into the proton magic nucleus ^{14}O has been observed with an intensity of 100% [22]. In this nucleus, the two protons occupy the $1d_{3/2}$ orbits above the $Z = 8$ closed shell, leading to $G = 4$. With the experimental decay energy of $Q_{2\mathrm{p}} = 1411 \pm 20$ keV [22], we obtained a folding factor of $\lambda = 0.662$ using the Bohr-Sommerfeld condition. The calculated half-life of the diproton emission is 6.5×10^{-20} s agreeing with the observed half-life of 9×10^{-21} s [22] within one order of magnitude. Another promising candidate for the diproton radioactivity is the drip-line nucleus ^{38}Ti decaying into the proton magic nucleus ^{36}Ca. The experiment has estimated a upper limit of 120 ns for the half-life of the diproton decay [25]. With the evaluated decay energy of $Q_{2\mathrm{p}} = 960 \pm 260$ keV from Ref. [22], we have determined a folding factor of $\lambda = 0.75$ at $G = 6$, resulting in the calculated diproton half-life of $T_{1/2} = 120$ ns. If the evaluated error of ± 260 keV in the Q value is considered, our calculations lead to a range of $T_{1/2} = 0.26$ ns to 1.7×10^6 ns.

In summary, an uniform folded cluster potential has been suggested for the WKB calculations of various cluster decays in atomic nuclei. The folded potential is constructed by nucleon potentials obtained from the self-consistent Skyrme-Hartree-Fock model, with the folding factor determined using the Bohr-Sommerfeld quantization condition combined with the Wildermuth condition. This leads to the consistent descriptions of the cluster decay and shell structure of a nucleus. The folded α-nucleus potential gives a reasonable volume integral, agreeing well with the experimental data obtained from α-scattering measurements. No adjustable parameter has been involved in cluster decay calculations, which is particularly useful for exotic nuclei since their properties are largely unknown experimentally. We have investigated the α-particle and heavier-cluster decays of trans-^{100}Sn and trans-^{208}Pb nuclei, and the diproton emission from the proton drip-line nucleus ^{16}Ne above the magic ^{14}O. The calculated half-lives agree with experimental data within a factor of ≈ 2 for α decays, and within one order of magnitude for heavier-cluster and diproton decays. We have also predicted the half-lives of possible α decays in the unknown drip-line nuclei ^{104}Te and ^{112}Ba, the expected ^{12}C emission from ^{114,112}Ba, and the diproton decay of ^{38}Ti. In our calculations, deformation effects on cluster radioactivities have not been taken into account,

which will be discussed in our other works.

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